

Exercise 1 Find e-values & e-vectors for A

$$A = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 6 & 4 \\ 0 & 3 & 2 \end{pmatrix}$$

► Charac. Polynomial $P(\lambda) = |\lambda I - A|$

$$\begin{vmatrix} \lambda & 0 & 0 \\ 0 & \lambda & 0 \\ 0 & 0 & \lambda \end{vmatrix} - \begin{vmatrix} 1 & 2 & 1 \\ 0 & 6 & 4 \\ 0 & 3 & 2 \end{vmatrix} = \begin{vmatrix} (\lambda-1) & -2 & -1 \\ 0 & (\lambda-6) & -4 \\ 0 & -3 & (\lambda-2) \end{vmatrix}$$

$$= (\lambda-1) [(\lambda-6)(\lambda-2) - 12] - (-2) [0-0] + (-1) [0-0]$$

$$= (\lambda-1) [\lambda^2 - 2\lambda - 6\lambda + 12 - 12] + 0$$

$$P(\lambda) = (\lambda-1)(\lambda^2 - 8\lambda) \longrightarrow \lambda^3 - 9\lambda^2 + 8\lambda \quad \textcircled{1}$$

► $0 = (\lambda-1)\lambda(\lambda-8) \longrightarrow \lambda_1 = 0, \lambda_2 = 1, \lambda_3 = 8$
(eigenvalues)

► Eigen vectors

• $\lambda_1 = 0$ $\textcircled{1}$
 $(\lambda I - A)x = 0$

$$\begin{pmatrix} (\lambda-1) & -2 & -1 \\ 0 & (\lambda-6) & -4 \\ 0 & -3 & (\lambda-2) \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\begin{pmatrix} -1 & -2 & -1 \\ 0 & -6 & -4 \\ 0 & -3 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Row Reduce

$$\begin{array}{l} R_3 \leftrightarrow \frac{1}{2} R_2 \\ -R_1 \end{array} \longrightarrow \begin{pmatrix} 1 & 2 & 1 \\ 0 & -6 & -4 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{-\frac{1}{6} R_2} \begin{pmatrix} 1 & 2 & 1 \\ 0 & 1 & \frac{2}{3} \\ 0 & 0 & 0 \end{pmatrix}$$

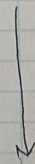
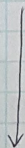
System of equations

$$\begin{cases} x_1 + 2x_2 + x_3 = 0 \\ 0 + x_2 + \frac{2}{3}x_3 = 0 \end{cases} \quad \text{set } x_3 = t$$

$$x_2 = -\frac{2}{3}t$$

$$x_1 = -\frac{2}{3}t + t = \frac{1}{3}t$$

$$v_1 = \begin{pmatrix} \frac{1}{3}t \\ -\frac{2}{3}t \\ t \end{pmatrix} \rightsquigarrow \begin{pmatrix} 1 \\ -2 \\ 3 \end{pmatrix} t \quad \text{b/c multiples}$$



• $\lambda_2 = 1$

(1)
$$\begin{pmatrix} 0 & -2 & -1 \\ 0 & -5 & -4 \\ 0 & -3 & -1 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

System of equations

~~$$\begin{cases} 0 + x_2 - x_3 = 0 \\ 0 + x_2 - 4x_3 = 0 \\ 0 + -3x_2 - x_3 = 0 \end{cases}$$~~

Row Reduce

$$\begin{array}{l} R_2 - (5/2)R_1 \\ R_3 - (3/2)R_1 \end{array} \rightarrow \begin{pmatrix} 0 & -2 & -1 \\ 0 & 0 & 3/2 \\ 0 & 0 & -1/2 \end{pmatrix} \xrightarrow{\begin{array}{l} -R_1/2 \\ R_3 + (1/3)R_2 \end{array}}$$

$$\begin{array}{l} R_1 - (1/3)R_2 \\ (2/3)R_2 \end{array} \rightarrow \begin{pmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{pmatrix}$$

System of equations

$$\begin{cases} 0 + x_2 + 0 = 0 & x_3 = t \\ 0 + 0 + x_3 = 0 & x_1 = t \\ 0 + 0 + 0 = 0 & x_2 = 0 \end{cases}$$

$$V_2 = \begin{pmatrix} t \\ 0 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ 0 \end{pmatrix} t$$

• $\lambda_3 = 0$

(1)
$$\begin{pmatrix} 7 & -2 & -1 \\ 0 & 2 & -4 \\ 0 & -3 & 6 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

Row Reduce

$$\begin{array}{l} R_3 + (3/2)R_2 \\ 1/7 R_1 \end{array} \rightarrow \begin{pmatrix} 1 & -2/7 & -1/7 \\ 0 & 2 & -4 \\ 0 & 0 & 0 \end{pmatrix} \xrightarrow{\begin{array}{l} R_1 + (1/7)R_2 \\ 1/2 R_2 \end{array}}$$

System of equations

$$\begin{cases} x_1 + 0 - (5/7)x_3 = 0 & x_3 = t \\ 0 + x_2 - 2x_3 = 0 & x_2 = 2t \\ 0 + 0 + 0 = 0 & \end{cases}$$

$$x_1 = (5/7)t$$

$$V_3 = \begin{pmatrix} 5/7 t \\ 2t \\ t \end{pmatrix} = \begin{pmatrix} 5/7 \\ 2 \\ 1 \end{pmatrix} t$$

Exercise 2 Find P & D . Check $PD=AP$ $\{D=P^{-1}AP\}$

• from eigenvectors,

$$P = \begin{pmatrix} 1/3 & 1 & 5/7 \\ -2/3 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix} \textcircled{1}$$

• from eigenvalues,

$$D = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{pmatrix} \textcircled{1}$$

• Check $PD=AP$

$$PD = \begin{pmatrix} 1/3 & 1 & 5/7 \\ -2/3 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix} \begin{pmatrix} 0 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 8 \end{pmatrix} = \begin{pmatrix} (0+0+0) & (0+1+0) & (0+0+40/7) \\ (0+0+0) & (0+0+0) & (0+0+16) \\ (0+0+0) & (0+0+0) & (0+0+8) \end{pmatrix}$$

$$\textcircled{1} = \begin{pmatrix} 0 & 1 & 40/7 \\ 0 & 0 & 16 \\ 0 & 0 & 8 \end{pmatrix} \checkmark$$

~~$$AP = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 6 & 4 \\ 0 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1/3 & 1 & 5/7 \\ -2/3 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} (1+0+0) & (0+2+0) & (0+0+5) \\ (0+0+0) & (0+6+0) & (0+0+10) \\ (0+0+0) & (0+3+0) & (0+0+2) \end{pmatrix}$$~~

$$AP = \begin{pmatrix} 1 & 2 & 1 \\ 0 & 6 & 4 \\ 0 & 3 & 2 \end{pmatrix} \begin{pmatrix} 1/3 & 1 & 5/7 \\ -2/3 & 0 & 2 \\ 1 & 0 & 1 \end{pmatrix} = \begin{pmatrix} (1/3 - 4/3 + 1) & (1+0+0) & (5/7 + 4 + 1) \\ (0 - 4 + 4) & (0+0+0) & (0 + 12 + 4) \\ (0 + 2 + 2) & (0+0+0) & (0+6+2) \end{pmatrix}$$

$$\textcircled{1} = \begin{pmatrix} 0 & 1 & 40/7 \\ 0 & 0 & 16 \\ 0 & 0 & 8 \end{pmatrix} \checkmark$$

$\therefore PD=AP$